Warm-up!

A) Solve over the interval $0 \leq \theta < 2\pi$:
$6\sin\theta + 2 = -1$

$\frac{\pi}{6} = \arcsin\left(\frac{1}{2}\right)$

$6\sin\theta = -3$

$\sin\theta = -\frac{1}{2}$

$\pi/6$

$\pi + \frac{\pi}{6} = \frac{7\pi}{6}$

$\pi/6$

$2\pi - \frac{\pi}{6}$

$\frac{11\pi}{6}$

B) Solve over the interval $0 \leq \theta < 2\pi$:
tan

$1 + \tan^2\theta = \sec^2\theta$

$\tan^2\theta = \sec^2\theta - 1$

$\sec^2\theta - 1 - \sec\theta - 1 = 0$

$\sec^2\theta - \sec\theta - 2 = 0$

$(\sec\theta - 2)(\sec\theta + 1) = 0$

$\sec\theta = 2$ $\qquad$ $\sec\theta = -1$

$\cos\theta = \frac{1}{2}$ $\qquad$ $\cos\theta = -1$

$\pi/3$ $\qquad$ $\pi/3$

$\theta = \frac{\pi}{3}$ $\qquad$ $\theta = \frac{5\pi}{3}$ $\qquad$ $\theta = \pi$

C) I'm thinking of a number. When squared, my number is . Of what number am I thinking?

± 5

$\sqrt{25} = 5$

$5^2 = 25$

Periodic Graphs Day 7:
- (3.9) Inverse Trigonometric Functions
- (3.10) Solving Trig Equations
- Solving Inverse Trig Equations

Homework: Inverse Trig Functions

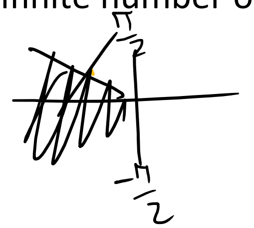
Quiz The Class After Next:
- An FRQ #3 style question
- Graphs of tan/cot/sec/csc
- Solving Trig Equations Using Inverse Trig

$\arcsin(x)$

$\frac{\pi}{6} \to \boxed{\sin} \to \frac{1}{2}$ $\qquad$ $\frac{1}{2} \to \boxed{\text{arcsin}} \to \frac{\pi}{6}$

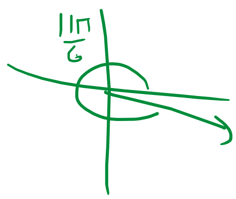
$\sqrt{25}$ asks the question "What positive number makes 25 when you square it?"
In other words, $x^2 = 25$ has two solutions, but $\sqrt{x}$ is only asking for the solution in $[0, \infty)$

$\arcsin\left(\frac{1}{2}\right)$ asks the question "What angle in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ makes $\frac{1}{2}$ when you take the sine?"
In other words, $\sin\theta = \frac{1}{2}$ has an infinite number of solutions, but $\arcsin\left(\frac{1}{2}\right)$ is only asking for the solution in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$



Practice:

$\arcsin\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$

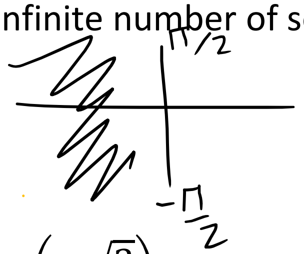


$\arcsin\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$

$\arcsin(-1) = -\frac{\pi}{2}$

$\arctan(x)$

$\arctan\left(\sqrt{3}\right)$ asks the question "What angle in $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ makes $\sqrt{3}$ when you take the tangent?" In other words, $\tan\theta = \sqrt{3}$ has an infinite number of solutions, but $\arctan\left(\sqrt{3}\right)$ is only asking for the solution in $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$



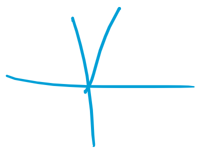
Practice:

$\arctan(1) = \frac{\pi}{4}$ $\arctan\left(-\frac{\sqrt{3}}{3}\right) = -\frac{\pi}{6}$ $\arctan(0) = 0$

$\tan\left(\frac{\pi}{4}\right) = 1$

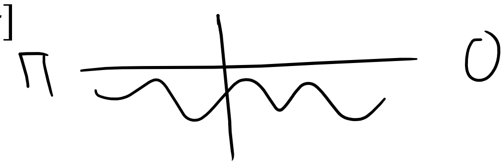
$\arctan(1) = \frac{\pi}{4}$

$\tan\left(\frac{\pi}{3}\right) = \sqrt{3}$ $\tan\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{3}$



arccos(x)

$\arccos\left(-\frac{\sqrt{3}}{2}\right)$ asks the question "What angle in $[0, \pi]$ makes $-\frac{\sqrt{3}}{2}$ when you take the cosine?" In other words, $\cos\theta = -\frac{\sqrt{3}}{2}$ has an infinite number of solutions, but $\arccos\left(-\frac{\sqrt{3}}{2}\right)$ is only asking for the solution in $[0, \pi]$



Practice:

$\arccos(1) = 0$

$\arccos\left(-\frac{\sqrt{2}}{2}\right) = \frac{3\pi}{4}$

$\arccos\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$

$\cos\theta = -\frac{1}{2}$

$\theta = \frac{2\pi}{3}$ $\theta = \frac{4\pi}{3}$

Some things to note

The range of each of these three inverse trig functions is different. Each includes the first quadrant and a neighboring quadrant.

$\arcsin(x)$ Range: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
$\arctan(x)$ Range: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
$\arccos(x)$ Range: $[0, \pi]$

Sometimes $\arcsin(x)$ is written as $\sin^{-1}(x)$
Sometimes $\arctan(x)$ is written as $\tan^{-1}(x)$
Sometimes $\arccos(x)$ is written as $\cos^{-1}(x)$

Trig functions turn angles into ratios. Inverse trig functions turn ratios back into angles.

Find the angle for each inverse function (without a calculator)

A) $\tan^{-1}\left(\frac{\sqrt{3}}{3}\right) = \frac{\pi}{6}$

B) $\arcsin\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3}$

C) $\cos^{-1}\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$

D) $\arctan(-1) = -\frac{\pi}{4}$

Game version of this:

$\arccos\left(-\frac{\sqrt{2}}{2}\right) = \frac{3\pi}{4}$

Find the angle for each inverse function (without a calculator)

E) $\arccos\left(-\frac{1}{2}\right)$

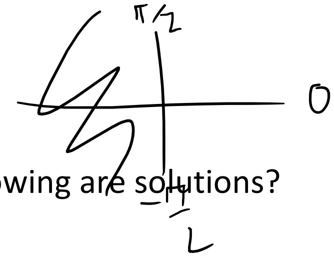
F) $\arccos(0)$

G) $\sin^{-1}(1)$

H) $\arctan\left(-\sqrt{3}\right)$

Game version of this:

Solve

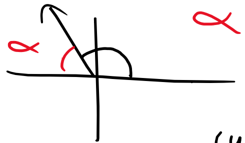


$$3\tan\theta + 5 = 1$$

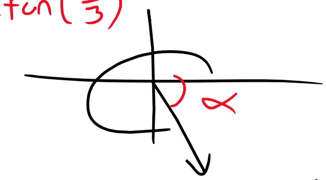
$3\tan\theta = -4$

$\tan\theta = -\frac{4}{3}$

$\alpha = \arctan\left(\frac{4}{3}\right)$



$\pi - \arctan\left(\frac{4}{3}\right)$



$2\pi - \arctan\left(\frac{4}{3}\right)$

Which of the following are solutions?

A) $\cancel{\arctan\left(\frac{4}{3}\right)}$

B) $\pi - \arctan\left(\frac{4}{3}\right)$

C) $\cancel{\pi + \arctan\left(\frac{4}{3}\right)}$

D) $2\pi - \arctan\left(\frac{4}{3}\right)$

E) $\cancel{2\pi + \arctan\left(\frac{4}{3}\right)}$

F) $\arctan\left(-\frac{4}{3}\right)$

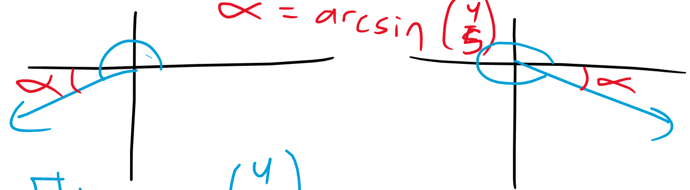
Solve

$$5\sin\theta + 6 = 2$$

$5\sin\theta = -4$

$\sin\theta = -\frac{4}{5}$

$\alpha = \arcsin\left(\frac{4}{5}\right)$



$\pi + \arcsin\left(\frac{4}{5}\right)$

$2\pi - \arcsin\left(\frac{4}{5}\right)$

$\parallel$

$\arcsin\left(-\frac{4}{5}\right)$

Which of the following are solutions?

A) $\sin^{-1}\left(\frac{4}{5}\right)$

B) $\sin^{-1}\left(-\frac{4}{5}\right)$

C) $2\pi - \sin^{-1}\left(\frac{4}{5}\right)$

D) $\pi - \sin^{-1}\left(\frac{4}{5}\right)$

E) $\pi + \sin^{-1}\left(\frac{4}{5}\right)$

Solve over the interval $0 \leq \theta < 2\pi$. Write unfamililar angles in terms of arccosine.

$$6\cos^2\theta + \cos\theta - 1 = 0$$

Find ALL solutions. Write unfamiliar solutions in terms of arccosine.

$$\sec^2 \theta - 4 \sec \theta - 5 = 0$$

Inverse Trig Equations

Step 1: Isolate the inverse trig function

Step 2: Apply the corresponding trig function

$$2\boxed{\arctan(3x-1)} = \frac{\pi}{2}$$

$$\arctan(3x-1) = \frac{\pi}{4}$$

$$\tan\left(\arctan(3x-1)\right) = \tan\left(\frac{\pi}{4}\right)$$

$$3x-1 = 1$$

$$3x = 2$$

$$x = \frac{2}{3}$$

Inverse Trig Equations

Let $p(x) = 3\sin^{-1}(\pi x) + \pi$

Find all input values in the domain of p that yield an output value of $\frac{3\pi}{2}$

$$\frac{3\pi}{2} = 3\sin^{-1}(\pi x) + \pi$$

$$\frac{\pi}{2} = 3\sin^{-1}(\pi x)$$

$$\frac{\pi}{6} = \sin^{-1}(\pi x)$$

$$\sin\left(\frac{\pi}{6}\right) = \sin\left(\sin^{-1}(\pi x)\right)$$

$$\frac{1}{2} = \pi x$$

$$\frac{1}{2\pi} = x$$

Inverse Trig Equations

Let $r(x) = \cos^{-1}(6x)$

Solve $r(x) = \frac{\pi}{3}$ for all values of x in the domain of r.

$$\frac{\pi}{3} = \arccos(6x)$$

$$\cos\left(\frac{\pi}{3}\right) = \cos\left(\arccos(6x)\right)$$

$$\frac{1}{2} = 6x$$

$$\frac{1}{12} = x$$

More Evaluating Practice

$\arctan(-1)$

$\arcsin\left(-\frac{\sqrt{2}}{2}\right)$

$\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$\cos^{-1}\left(-\frac{1}{2}\right)$

$\arccos(0)$

$\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$\ldots, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \ldots$

$\theta = \frac{\pi}{2} + \pi k$

tan( )

sec( )

$\theta = -\pi, 0, \pi, 2\pi, 3\pi, \ldots$

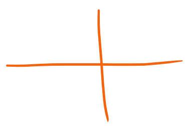
cot( )

$\theta = \pi k$

csc( )

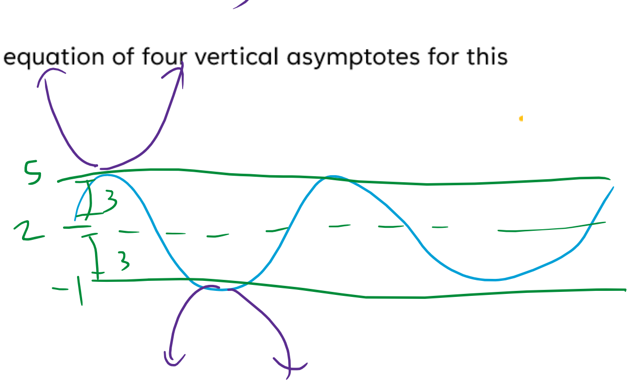
Range: $(-\infty, -1] \cup [5, \infty)$

7. Find the range of $f(x) = 3\sec\left(\boxed{\frac{1}{3}x}\right) + 2$. Then, find the equation of four vertical asymptotes for this function.



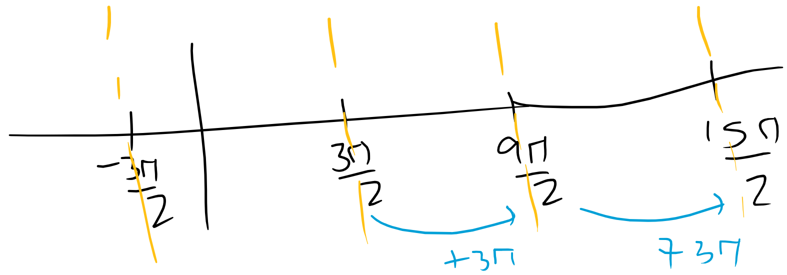
$3\cos\left(\frac{1}{3}x\right)+2$

Range: $[-1, 5]$



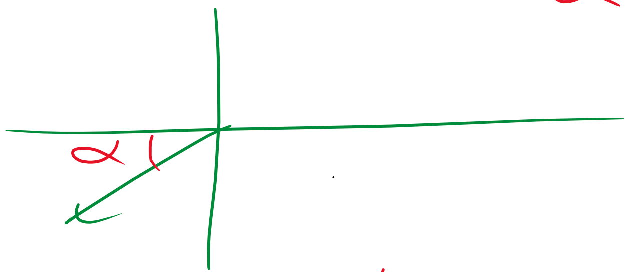
$\frac{1}{3}x = \frac{\pi}{2} + \pi k$

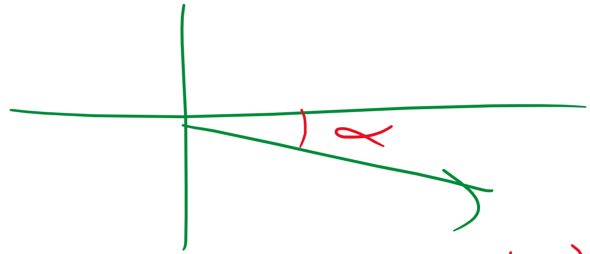
$x = \frac{3\pi}{2} + 3\pi k$



$$\sin\theta = -\frac{1}{2}$$

$$\alpha = \arcsin\left(\frac{1}{2}\right) = \frac{\pi}{6}$$





$\pi + \arcsin\left(\frac{1}{2}\right)$

$2\pi - \arcsin\left(\frac{1}{2}\right)$